

# Implications of observed neutrinoless double beta decay

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## Abstract

Recently a positive indication of the neutrinoless double beta decay has been announced. We study the implications of this result taking into consideration earlier results on atmospheric neutrinos and solar neutrinos. We also include in our discussions the recent results from SNO and K2K. We point out that on the confidence level given for the double beta signal, the neutrino mass matrices are now highly constrained. All models predicting Dirac masses are ruled out and leptogenesis becomes a natural choice. Only the degenerate and the inverted hierarchical solutions are allowed for the three generation Majorana neutrinos. In both these cases we find that the radiative corrections destabilize the solutions and the LOW, VO and Just So solutions of the solar neutrinos are ruled out. For the four generation case only the inverted hierarchical scenario is allowed.

Recent evidence of non-zero neutrino mass in the atmospheric neutrino anomaly [1], strong constraints from the solar neutrinos [2] and some positive indications from Laboratory experiments [3, 4] have already restricted the possible neutrino mass matrices to only a few possible choices [5]. The atmospheric neutrino result is consistent with an oscillation between the  $\nu_\mu$  and  $\nu_\tau$  with maximal mixing.<sup>1</sup> There are several allowed regions in the parameter space of the mass squared differences and mixing angles of the neutrinos, which can explain the solar neutrino problem [8, 9]. But none of these results could tell us about the absolute value of the neutrino mass and whether the neutrinos are Majorana or Dirac particles, *i.e.*, whether the neutrino mass conserves lepton number or not.

A positive signal of the neutrinoless double beta decay has recently been announced [10]. The  $0\nu\beta\beta$  decay describes a process in which two electrons are released (with lepton number two) without any associated antineutrinos, and hence lepton number is broken [11]. This process can be mediated by a light

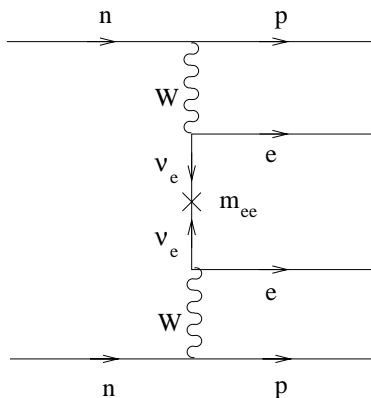


Figure 1: Diagram mediating neutrinoless double beta decay.

virtual electron neutrino  $\nu_e$  (as shown in figure 1), whose effective Majorana mass

$$\mathcal{L}_{Maj} = m_{ee}\nu_e\nu_e$$

breaks the lepton number. This physical electron neutrino state  $\nu_e$  is the neutrino which couples to the physical electron. So,  $m_{ee}$  is the (11) element of the neutrino mass matrix  $M_{\alpha\beta}^\nu$ , ( $\alpha = e, \mu, \tau$ ) in the basis in which the charged lepton mass matrix is diagonal. The physical state  $|\nu_\alpha\rangle$  is related to the mass

<sup>1</sup>Even though the  $L/E$  flatness of the electron-like event ratio in the full three-flavor framework suggests a bi-maximal mixing [6], here we shall restrict to the simplest scenario in which one explains the atmospheric neutrino anomaly via  $\nu_\mu \leftrightarrow \nu_\tau$  oscillations. In addition, in our analysis we shall not incorporate the quantum gravity effects that may induce additional modifications to neutrino oscillations [7].

eigenstates  $\nu_i$  (with eigenvalues  $m_i$ ,  $i = 1, 2, 3$ ) by the mixing matrix  $U_{\alpha i}$  by

$$|\nu_\alpha\rangle = U_{\alpha i}|\nu_i\rangle \quad (1)$$

so that the mass matrix in the flavour basis is related to the diagonal mass matrix through

$$M_{\alpha\beta}^\nu = U_{\alpha i} M_{ij}^{\text{diag}} U_{\beta j}^T \quad (2)$$

where  $M_{ij}^{\text{diag}} = m_i \delta_{ij}$ . The  $0\nu\beta\beta$  decay rate depends on this effective Majorana mass [13, 14]

$$m_{ee} = M_{ee}^\nu = \sum_i |U_{ei}|^2 m_i. \quad (3)$$

Taking care of the nuclear matrix elements and other factors, the present signal for the  $0\nu\beta\beta$  decay amounts to an effective Majorana mass of the electron neutrino in the range of

$$m_{ee} = (0.05 - 0.86) \text{ eV} \quad \text{at 95\% c.l.} \quad (4)$$

with a best value of 0.4 eV. This is the first indication of lepton number violation in nature and the fact that Majorana fermions can exist in nature. This result establishes that a fermion can be its own antiparticle, violating lepton number by two units. There are several other consequences of this new result for physics beyond the standard model [12]. Here we study some of the implications for the neutrino masses.

In the standard model neutrinos are massless. One may extend the model with one singlet right-handed neutrino  $N_{aR}$  per generation. Then the interaction  $h_{ia} \bar{\ell}_{iL} N_{aR} \phi$  can give a Dirac mass, where  $h_{ia}$  are the Yukawa couplings,  $\ell_{iL}$  are left-handed leptons and  $\phi$  is the usual Higgs doublet. For the neutrino mass to be of the order of eV, the Yukawa couplings have to be extremely small,  $h \sim 10^{-12}$ . Although such small numbers are not natural, this is surely a possibility. If this is the only source of neutrino mass, then the contribution to the neutrinoless double beta decay will come from two diagrams, one from an exchange of  $\nu_e$  and the other from an exchange of  $N_R$ . These two diagrams will cancel each other exactly [14]. A naive way of understanding this result is to consider the lepton number. Since  $\ell_{iL}$  and  $N_{aR}$  both carry lepton number one, the above Dirac mass term does not violate lepton number. So this term cannot allow a lepton number violating process like  $0\nu\beta\beta$  decay. Thus the new result rules out this unnatural possibility of Dirac neutrinos with unnaturally tiny Yukawa couplings completely.

A natural choice for small neutrino mass is to consider an effective five dimensional operator [15] with only the left-handed leptons

$$\mathcal{L}_{eff} = \frac{f_{ij}}{M} \ell_{iL} \ell_{jL} \phi \phi. \quad (5)$$

For any large lepton number violating scale  $M$ , this operator would then give a very small Majorana mass to the neutrinos. Within the context of the standard model this is the only effective term allowed, which can give a neutrino mass. However, since this is not a renormalizable term, this term cannot originate from the standard model alone. There has to be some higher theory at the cut-off scale  $M$ , which is the lepton number violating scale in this case. So, this already indicates the nature of physics beyond the standard model.

This operator has several possible realizations. In the see-saw mechanism [16] one breaks lepton number with a Majorana mass of the right-handed neutrinos

$$\mathcal{L}_N = M_{Nab} N_{aR} N_{bR}$$

The interplay of the Dirac mass term and this term then induces a lepton number violating effective neutrino mass term

$$\mathcal{L}_\nu = \frac{h_{ai} h_{bj}^T \langle \phi \rangle^2}{M_{Nab}} \nu_{iL} \nu_{jL},$$

which can give rise to the neutrinoless double beta decay.

In another realization of the effective dimension five operator a triplet Higgs is introduced [17, 18]. In one version of the model [17], lepton number was broken with a vacuum expectation value ( $vev$ ) of the triplet Higgs, resulting in a Majoron. These scenarios are ruled out from the Z-width at LEP. A newer version of the model now breaks lepton number explicitly at a very high scale  $M \sim M_\xi \sim \mu$  through the couplings of the triplet Higgs  $\xi$

$$\mathcal{L}_\xi = M_\xi^2 \xi^\dagger \xi + f_{ij} \ell_{iL} \ell_{jL} \xi + \mu \xi^\dagger \phi \phi.$$

The minimization of the complete potential for  $\mu \neq 0$  then gives a small  $vev$  to the triplet Higgs  $\langle \xi \rangle \sim \frac{\mu \langle \phi \rangle^2}{M_\xi^2}$ , which generates a Majorana mass of the neutrino.

If one extends the standard model to a larger left-right symmetric model, then in the left-right symmetric model neutrinos can acquire mass from both the see-saw mechanism and the triplet Higgs mechanism [19]. Then there are radiative mechanisms [20] with a low lepton number violating scale.

If we now consider the dimension five operator, the effective Majorana neutrino mass is  $m_{ee} \sim f_{11} \langle \phi \rangle^2 / M$ . For  $f_{11} \sim 0.01$  the present result of  $0\nu\beta\beta$  decay implies a lepton number violating scale of  $M \sim 10^{10}$  GeV. This lepton number violating scale can then address another important question of the baryon asymmetry of the universe [21, 22]. In both the see-saw [21] and the triplet Higgs models [18], with lepton number violating scale it is possible to generate a lepton asymmetry of the universe.

In the see-saw mechanism, when the right-handed neutrinos decay into leptons and antileptons, lepton number is violated at the temperature  $T = M_{Nab}$ . The Majorana phases of the right-handed neutrinos give enough CP violation

in these decays and the out-of-equilibrium condition is naturally satisfied at this scale. This would then generate a lepton asymmetry of the universe. In the triplet Higgs models the decays of the triplet Higgs to Higgs doublets and the leptons violate lepton number at the temperature  $T = M_\xi$ . The coupling of the triplet Higgs,  $f_{ij}$  and  $\mu$ , contains the required CP violation. Before the electroweak phase transition this lepton asymmetry would then get converted to the baryon asymmetry of the universe in the presence of the sphalerons [23].

Recently a new possibility of TeV scale gravity with extra dimensions has become very promising phenomenologically [24]. In these models all the standard model particles are confined only in our 4-dimensional world, while gravity can propagate in all the directions, including the extra dimensions forming most of the bulk of space-time. Although gravity is strong in the extra dimensions, the small overlap of our world with the extra dimensions makes gravity weak in our world. In these models of extra dimensions one includes a right-handed neutrino field in the bulk [25]. The overlap of the right-handed neutrino in our world will then be small and this can make the Yukawa coupling  $h_{ia}$  in  $h_{ia}\bar{\ell}_{iL}N_{aR}\phi$  naturally small, so that now there can be a very small Dirac mass of the neutrinos. The Yukawa coupling  $h_{ia}$  is naturally suppressed by the volume of the extra dimensions. There are several new phenomena associated with these scenarios including new predictions for neutrino oscillations [25]. All these models are now ruled out by the new observation.

The  $0\nu\beta\beta$  decay would now allow only a few possible choices for the neutrino mass in these theories of large extra dimensions [26, 27]. In one possibility [26] lepton number is broken in a distant brane and the effect shines in our world to give a small lepton number violating coupling of a Triplet Higgs scalar, which then generates a small neutrino mass. In this scenario the triplet Higgs will have definite same sign dilepton signals in the next generation colliders, which should be observed [26]. Of course one may consider an effective operator in the higher dimensions and can consider a Majorana mass, or break lepton number in the bulk to generate Majorana masses [27], details of realization of such models are yet to be studied.

Given the model of generating a small Majorana mass, the exact structure of the mass matrix may be such that there are two contributions from different mass eigenstates, which cancel each other forbidding  $0\nu\beta\beta$  decay. Thus the present observation of the  $0\nu\beta\beta$  decay would rule out a class of models where this cancellation takes place partially or fully. Moreover constraints from the atmospheric and solar neutrinos and Lab limits could be used in conjunction with the present limit on  $0\nu\beta\beta$  decay to discriminate some of the possible mass spectra following the analysis of [28] and with one new consideration that in some cases electroweak radiative corrections destabilize the LOW, VO and Just So solutions of the solar neutrinos.

Taking the latest analysis of the solar neutrino data, including SNO, eight solutions are allowed [9]. For three generation of neutrinos the solutions are categorised as (LMA) large mixing angle MSW solution; (SMA) small mixing

Table 1: Allowed neutrino mass squared differences ( $\Delta m^2$ ) and mixing angles ( $\sin^2 2\theta$ ) from solar and atmospheric neutrino results according to [9, 2, 8, 1, 3]. All masses are in eV. The last column shows if any particular solution is allowed by the new result from  $0\nu\beta\beta$  decay and our present analysis.

Solution	$\Delta m^2$	$\sin^2 2\theta$	$[(\beta\beta)_{0\nu}]$
Three generation Solar neutrino solutions			
SMA	$(4 - 9) \times 10^{-6}$	$(.0008 - .008)$	Allowed
LMA	$(2 - 20) \times 10^{-5}$	$(0.3 - 0.93)$	Allowed
LOW	$(6 - 20) \times 10^{-8}$	$(0.89 - 1)$	Not Allowed
VO	$10^{-10}$	$(0.7 - 0.95)$	Not allowed
Just So	$(5 - 8) \times 10^{-12}$	$(0.89 - 1)$	Not allowed
Solar neutrino solutions with sterile neutrino			
SMA	$(3 - 8) \times 10^{-6}$	$(.0006 - .008)$	Allowed
VO	$1.4 \times 10^{-10}$	$(0.7 - 0.9)$	Allowed
Just So	$(6 - 8) \times 10^{-12}$	$(0.82 - 1)$	Allowed
Atmospheric neutrino			
$\nu_\mu \rightarrow \nu_\tau$	$(1.8 - 4.0) \times 10^{-3}$	$(0.87 - 1.0)$	Consistent

angle MSW solution; (LOW) low probability low mass solution; (VO) vacuum oscillation solutions; (Just So) very low mass squared difference vacuum oscillation solutions. For our analysis, the three solutions LOW, VO and Just So, will not make much difference, so we call them together VAC solution. With sterile neutrinos only the SMA, VO and Just So solutions are allowed. The atmospheric neutrino anomaly and the initial results from K2K determine the  $\nu_\mu \rightarrow \nu_\tau$  mixing and mass squared difference [1, 3]. These results are summarized in table 1. Several of these solutions will now be ruled out by the new neutrinoless double beta decay result as mentioned in table 1.

First consider only three generations of neutrinos. The mixing matrix of equation [1] may be parametrized as

$$U = \begin{pmatrix} c_1 c_3 & -s_1 c_3 & -s_3 \\ s_1 c_2 - c_1 s_2 s_3 & c_1 c_2 + s_1 s_2 s_3 & -c_3 s_2 \\ s_1 s_2 + c_1 c_2 s_3 & c_1 s_2 - s_2 c_2 s_3 & c_2 c_3 \end{pmatrix}, \quad (6)$$

where,  $s_i = \sin \theta_i$  and  $c_i = \cos \theta_i$ .  $s_2$  represents  $\nu_\mu - \nu_\tau$  mixing and is determined by the atmospheric neutrino anomaly,  $s_3$  gives  $\nu_e - \nu_\tau$  mixing and is constrained by the CHOOZ result and  $s_1$  is related to the solar neutrinos. Then the contribution to the  $0\nu\beta\beta$  decay is given by

$$m_{ee} = \sum_i |U_{ei}|^2 m_i = m_1 c_1^2 c_3^2 + m_2 s_1^2 c_3^2 + m_3 s_3^2. \quad (7)$$

$m_i$  are complex and contain the Majorana phases, which can contribute to the lepton number violating  $0\nu\beta\beta$  decay. The phase in the mixing matrix  $U$  may only show up in the neutrino oscillation experiments and does not contribute to the lepton number violating  $0\nu\beta\beta$  decay.

The contributions of the different mass patterns to the  $0\nu\beta\beta$  decay may now be estimated for the allowed values of the mixing angles ( $s_2$  and  $s_3$ ) and the mass eigenvalues ( $m_1$ ,  $m_2$  and  $m_3$ ). We allow possible variation of the phases in the masses to check for cancellation. This gives a range for  $m_{ee}$ , which is similar to the values obtained in an earlier analysis [28]. There are some changes, which come due to the new input from SNO. Since the mixing angle  $s_2$  does not enter the expression [equation (7)], new results from K2K do not affect the analysis. In addition we consider electroweak radiative corrections and point out that they may destabilize the very small mass differences required for some of the solar neutrino solutions.

The different three generation models of the neutrino masses may be classified as:

**Hierarchical** The masses satisfy a hierarchical pattern

$$m_1 \ll m_2 \ll m_3,$$

$$\text{so that } m_3 = m_{atm} = \sqrt{\Delta m_{atm}^2} \quad \text{and} \quad m_2 = m_{sol} = \sqrt{\Delta m_{sol}^2}.$$

**Degenerate** All the three masses are of the same order

$$m_1 \approx m_2 \approx m_3 = m_0.$$

Atmospheric neutrinos require  $m_0 > 0.042$  eV, hot dark matter prefers  $m_0 \sim \text{eV}$ , but the neutrinoless double beta decay imposes  $m_0 < 0.8$  eV. The mass squared differences are much smaller

$$\Delta m_{12}^2 = \Delta m_{sol}^2 \quad \text{and} \quad \Delta m_{23}^2 = \Delta m_{atm}^2$$

**Partially Degenerate** The lighter masses are almost degenerate and their mass squared difference explains solar neutrinos

$$m_1 \approx m_2 < m_3 \approx m_{atm} = \sqrt{\Delta m_{atm}^2},$$

with  $\Delta m_{12} \approx \Delta m_{sol}$ .

**Inverted Hierarchical**  $\nu_e$  is part of the heavier states and the mass splitting between  $m_2$  and  $m_3$  explains the solar neutrino puzzle

$$m_1^2 \ll m_3^2 \approx m_2^2 = \Delta m_{atm}^2 \quad \text{and} \quad \Delta m_{23}^2 \approx \Delta m_{sol}^2.$$

In the hierarchical scenario for SMA solution the largest contribution to  $0\nu\beta\beta$  decay comes from  $m_3$  inspite of the small mixing angle  $s_3$  constrained by CHOOZ, so that  $m_3 s_3^2 < 0.002$  eV. For the LMA solution there is another contribution from  $m_2 \sim m_{sol} \sim 0.005$ , which is comparable and bounded by  $0.0004 < m_2 \sin^2 \theta < 0.0015$ . Since there is no lower bound on  $m_3$  contribution, both these contributions can cancel each other and again there is no lower bound. The upper bound is now  $m_{ee} < 0.0035$ . Thus the contribution to the  $0\nu\beta\beta$  decay is much smaller than that allowed by the present observation for all the hierarchical solutions to the solar neutrino problem. Another possibility (triple) with hierarchical mass matrix is to consider all elements of the mixing matrix to be equal, which has a definite prediction for the  $0\nu\beta\beta$  decay  $m_{ee} \sim 0.02$  and is also ruled out. For the different solutions of the hierarchical scenario, the predictions differ slightly, but all of them fall below the allowed region. This is shown in the summary figure 2.

For partially degenerate neutrino masses,  $0.005 \text{ eV} < m_1 < 0.042 \text{ eV}$  restricts the amount of  $0\nu\beta\beta$  decay. For SMA the main contribution comes from  $m_1$  and hence  $m_{ee}$  is bounded by the value of  $m_1$ . For the LMA solar neutrino solution the contribution from  $m_2$  is comparable but smaller, so considering the bound on the mixing angle, the bound becomes  $0.042 > m_{ee} > 0.0015$ . For the VAC solutions the mixing angle could be maximal and the contributions from  $m_1$  and  $m_2$  can cancel, so there is no lower bound. In all the cases the prediction for  $0\nu\beta\beta$  decay comes out to be smaller than the presently acceptable range for all the solutions of the solar neutrinos and hence all these solutions are ruled out. In ref. [10] the authors included the transition region (which is  $m_1 \approx m_2 \approx m_3$ ) in their definition of the partial degenerate solution. As a result they find an overlap of the partial degenerate solution with the present result of  $0\nu\beta\beta$  decay. But we have included the transition region in the degenerate mass spectrum and defined out partially degenerate solution as  $m_1 < m_3$ , so that the upper bound on  $m_1$  gives the bound  $m_{ee} < m_3 = m_{atm} = 0.042$ .

The degenerate mass spectrum can be realized, simultaneously providing a solution to the solar and atmospheric neutrinos, with  $0.042 \text{ eV} \leq m_1 \leq 1 \text{ eV}$ . In this case the contribution to  $m_{ee}$  comes mostly comes from  $m_1$  for the SMA. But for the LMA both  $m_1$  and  $m_2$  contribute and there is also a possibility of partial cancellation. Considering  $s_1 < 0.93$  the lower bound comes out to be  $m_1 c_1^2 - m_2 s_2^2 > 0.015$ . Both the SMA and the LMA solutions are thus allowed by the present observation of  $0\nu\beta\beta$  decay. But the VAC solutions for solar neutrinos, which require  $m_1^2 - m_2^2 < 10^{-8} \text{ eV}$ , are ruled out by the following argument.



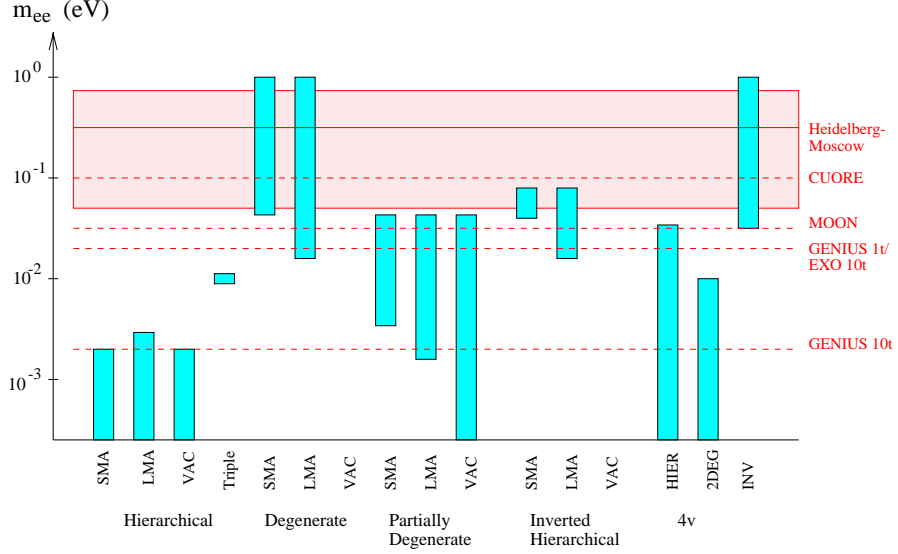


Figure 2: Contributions in different models to the neutrinoless double beta decay. The present result is given by the shaded region (the solid line denoting the best value), which allows only few of the models. The expected sensitivity for the CUORE [29], MOON [30] and the one ton and ten tons GENIUS [32] are given for comparison.

In the basis in which the charged lepton masses are diagonal, the diagonal terms in the neutrino mass matrices  $M_\nu$  will receive a contribution from higher order weak interaction corrections [33]. Consider for simplicity a mass matrix in the basis  $[\nu_e \ \nu_\tau]$

$$M_\nu = \begin{pmatrix} a & b \\ b & c \end{pmatrix}.$$

The degeneracy of the eigenvalues requires (1)  $a = c = 0$  or (2)  $a = -c = m_0$ . In case 1, the radiative correction does not change the eigenvalues and we have a Dirac neutrino. But in case 2, the radiative corrections break the degeneracy and give us a pseudo-Dirac neutrino [33]. The present result on  $0\nu\beta\beta$  decay requires  $a = m_{ee} \neq 0$ , so we are forced to consider case 2. In this case there is a mass splitting between the two degenerate states, given by the mass squared difference

$$(m_2^2 - m_1^2)_{rad} \sim \alpha_W m_0^2 \left( \frac{m_\tau^2 - m_e^2}{m_W^2} \right) \sim 1.7 \times 10^{-5} m_0^2 \text{ eV}^2. \quad (8)$$

$\alpha_W$  is the weak fine structure constant.

To explain the atmospheric neutrino anomaly, the degenerate mass should be  $m_0 > 0.042$  eV. Then the radiative corrections modify the tree level mass squared difference by  $(m_2^2 - m_1^2)_{rad} > 1.5 \times 10^{-8}$  eV<sup>2</sup>. With this correction it will not be possible to maintain the mass squared difference required for the VAC solutions and hence these VAC solutions are ruled out in the degenerate mass scenarios. Although we demonstrated with two generation example, this result is applicable to three generation case. When we discuss the specific textures we shall prove this generality.

This is true for the inverted hierarchy scenario as well. In this case the radiative correction comes out to be the same as above, with  $m_0 = m_2 \approx m_3$ . Thus the radiative correction would give a mass squared difference, larger than acceptable by the VAC solutions. So, even if there is a model to predict such small mass squared difference required by the VAC solutions, after including the weak radiative correction the model cannot maintain the required mass squared difference.

To demonstrate this result let us now work with one example. We assume a mass matrix in the basis with diagonal charged leptons

$$M_\nu = m_0 \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1/2 & 1/2 \\ 0 & 1/2 & 1/2 \end{pmatrix}. \quad (9)$$

which can be diagonalised to

$$M^{diag} = \text{Diag} \{m_3, m_2, m_1\} = m_0 \text{Diag} \{1, 1, 0\}.$$

This predicts maximal mixing between  $\nu_\mu$  and  $\nu_\tau$ . Inclusion of the radiative corrections would change this mass matrix to

$$M_\nu = m_0 \begin{pmatrix} 1 + \epsilon_e & 0 & 0 \\ 0 & (1 + \epsilon_\mu)/2 & 1/2 \\ 0 & 1/2 & (1 + \epsilon_\tau)/2 \end{pmatrix} \quad (10)$$

where  $\epsilon_i = \alpha_W(m_i^2/m_W^2)$ . After diagonalisation the new mass eigenvalues will become  $m_\nu^{diag} \sim \text{Diag } m_0 \{1 + \epsilon_e, 1 + \epsilon_\mu + \epsilon_\tau, \epsilon_\mu + \epsilon_\tau\}$ , so that the mass squared difference between the two degenerate states has become  $m_2^2 - m_3^2 \sim 1.7 \times 10^{-5} m_0^2$  eV<sup>2</sup>. For  $m_0 \sim m_{atm} > 0.042$  this is too large for the VAC solutions.

In this inverted hierarchical case the LMA and SMA solutions of solar neutrinos are still allowed by the present result of the  $0\nu\beta\beta$  decay. Since the heavier states  $\nu_2$  and  $\nu_3$  contain the  $\nu_e$ , they contribute to the  $0\nu\beta\beta$  decay. But the mass is now restricted by the solution to the atmospheric neutrino anomaly, since  $m_2 \sim m_{atm}$ , we get  $0.063 > m_{ee} > 0.042$ . For the LMA solution for solar neutrinos there can be cancellation but with the present limit [9] on mixing for the LMA solution the cancellation can only be partial and the bound is  $0.063 > m_{ee} > 0.015$ .

Thus for the three generations of neutrinos, the LOW, VO and the Just So solutions of the solar neutrinos are not allowed. The hierarchical and partially degenerate mass spectrum for all solutions of the solar neutrinos are ruled out. Only the degenerate and inverted hierarchical mass spectrum are allowed which can provide SMA and LMA solutions for solar neutrinos. There are some theoretical problems with these cases. If one starts from a grand unified theory and tries to evolve the Yukawa couplings in supersymmetric models, it becomes difficult to maintain the degeneracy of two or three states [34]. It has been pointed out that if there is some symmetry which protects some of the texture zeroes in the mass matrix, then these zeroes are protected against the renormalization group evolution [35]. There could be another possibility to this problem of protecting the degeneracy. In models of large extra dimensions the evolution of the Yukawa couplings can allow degenerate solutions.

Considering this model building point of view, it is convenient to study some textures of the neutrino mass matrix. Earlier they were considered to explain the atmospheric neutrino problem assuming that the mass squared difference required by the solar neutrino would come as perturbation to these texture mass matrices. Since the solar neutrino requires small mass squared difference, this assumption is justified.

In the degenerate and the inverted hierarchical scenarios, which are now allowed by the  $0\nu\beta\beta$  decay result, all the zeroth order neutrino mass matrices ( $M_\nu$ ) with texture zeroes can be listed [5]

$$\begin{aligned}
M_\nu^{B1} &= m_0 \begin{pmatrix} 1 & 0 & 0 \\ 0 & -1/2 & -1/2 \\ 0 & -1/2 & -1/2 \end{pmatrix} & M_\nu^{\text{diag}} &= \text{Diag} \{m_0, -m_0, 0\} \\
M_\nu^{B2} &= m_0 \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1/2 & 1/2 \\ 0 & 1/2 & 1/2 \end{pmatrix} & M_\nu^{\text{diag}} &= \text{Diag} \{m_0, m_0, 0\} \\
M_\nu^{C0} &= m_0 \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} & M_\nu^{\text{diag}} &= \text{Diag} \{m_0, m_0, m_0\} \\
M_\nu^{C1} &= m_0 \begin{pmatrix} -1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} & M_\nu^{\text{diag}} &= \text{Diag} \{-m_0, m_0, m_0\} \\
M_\nu^{C2} &= m_0 \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & -1 \\ 0 & -1 & 0 \end{pmatrix} & M_\nu^{\text{diag}} &= \text{Diag} \{m_0, -m_0, m_0\} \\
M_\nu^{C3} &= m_0 \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix} & M_\nu^{\text{diag}} &= \text{Diag} \{m_0, m_0, -m_0\}
\end{aligned}$$

where the  $B$  solutions are for inverted hierarchical scenario and  $C$  solutions are for degenerate cases. For  $B$  solutions,  $m_0^2 = \Delta m_{atm}^2$  and for  $C$  solutions  $m_0^2 > \Delta m_{atm}^2$ . In case  $C2$  and  $C3$  it may appear that the radiative corrections may be small. But in case  $C2$  it is not possible to have a solution to solar neutrinos with maximal mixing, so the vacuum solutions are not possible. This mass matrix allows only a small mixing angle solution.

Let us now discuss case  $C3$  in little details, which will demonstrate how the radiative correction destabilize the vacuum solutions for three generation case. Let us say  $m_0 \gg m_{atm}$ . To generate a mass difference  $a \sim O(m_{atm})$  between  $\nu_\mu$  and  $\nu_\tau$  (with  $m_{\nu_\mu} < m_{\nu_\tau}$ ) without disturbing the degeneracy between  $\nu_e$  and  $\nu_\tau$ , we need to introduce a term  $a$  in the (33) element of the mass matrix. This will introduce a mass difference between  $\nu_e$  and  $\nu_\tau$  radiatively, which is  $m_0 a \epsilon_\tau > 10^{-8}$  eV, destabilizing the VAC solutions in this case. Thus for all mass textures and degenerate solutions, the VAC solutions are ruled out.

In all textures the mixing for the atmospheric neutrinos is considered to be maximal,  $s_2 = c_2 = 1/\sqrt{2}$ . It is also assumed that the electron neutrino does not take part ( $s_3 = 0$ ) in atmospheric neutrino anomaly. Thus the mixing matrix can be parametrized only in terms of the solar neutrino mixing angle  $s = s_1$  ( $c = c_1$ )

$$U = \frac{1}{\sqrt{2}} \begin{pmatrix} \sqrt{2}c & -\sqrt{2}s & 0 \\ s & c & -1 \\ s & c & 1 \end{pmatrix}. \quad (11)$$

For complete solutions suitable small perturbations need to be added. But if at this level the (11) element vanishes, then with small perturbation it will not be possible to predict the required amount of  $0\nu\beta\beta$  decay. This criterion rules out several of the mass matrices with texture zeroes [5].

Let us now include sterile neutrinos in the discussion. The latest result from Super-Kamiokande rules out sterile neutrino mixing for only two generations [36]. In a more general analysis with four generations it has been shown that it is not ruled out, but the allowed parameter space is restricted [37]. On the other hand the main motivation of introducing the sterile neutrino is to explain the LSND result [4], which is in partial conflict with the KARMEN result [38]. Considering all this we shall not give much details with sterile neutrinos.

One possible mass spectrum could be hierarchical, where the sterile neutrino has a mass of the order of eV to explain LSND and the other neutrinos have similar structure as the three generation hierarchical case. The sterile neutrino mixes very weakly with other generations. So the contributions to the  $0\nu\beta\beta$  decay coming from all the states are small, and taking the maximum allowed values for the different mixing angles this scenario predicts  $m_{ee} < 0.03$  eV. This possibility is thus not allowed by the present  $0\nu\beta\beta$  decay result. Another possibility is that two of the heavier states are composed of  $\nu_\mu$  and  $\nu_\tau$ , whose mass difference explains the atmospheric neutrino problem.  $\nu_e$  is the lightest and its mixing with  $\nu_\mu$  explains LSND. This scenario is also ruled out, since it

predicts  $m_{ee} < 0.01$ .

The inverted hierarchical scenarios are not ruled out, where  $\nu_e$  and  $\nu_s$  [or  $\nu_\tau$ ] have mass to explain LSND and small mass squared difference to explain solar neutrinos. The reason is that the mass difference between  $\nu_\mu$  and the fourth neutrino is such that they explain the atmospheric neutrino problem. For large mixing there will be cancellation and the contribution to the  $0\nu\beta\beta$  decay will be small. So only a very restricted parameter space is now allowed by the new  $0\nu\beta\beta$  decay result.

In summary, we studied the implications of the first evidence for the neutrinoless double beta decay. This is the first evidence for lepton number violation and for a Majorana particle. So models of Dirac neutrinos are now ruled out and the models of Majorana neutrinos can generate a lepton asymmetry of the universe. The structure of the Majorana mass matrix also gets more constrained. For the three generation case the hierarchical and partially degenerate neutrino mass matrices are not allowed. Only the degenerate and inverted hierarchical models are allowed. Even in these two cases, if one considers electroweak radiative corrections, the small mass solutions of the solar neutrinos become difficult to accommodate. For the allowed scenarios possible texture mass matrices are mentioned. In the four generation scenario, only the inverted hierarchical models are allowed.

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